# STEMpunk's shooter, optimized with applied mathematics:

Ever wonder what the single greatest, most spectacular, ABSOLUTE BEST angle and velocity to shoot your game piece is?

Well, for us, it's 39 degrees at 29.53 feet per second.

If you want to optimize your robot like we did ours, I would encourage you to read on and find out how got these numbers.

A week into the build season our robot was well past the first basic stages of design. Our team, having decided on the layout of our drivetrain (utilizing an unusual application of the mechanum drive) and our shooter, (a virtually one piece rig functioning as an all in one, passing, shooting, ground/ air pickup mechanism) needed to know now, what results our shooter should produce to classify as a "good shooter".

These algorithms, if utilized properly, should afford most teams with the capability to find that one combination of angle and velocity giving them the single greatest, most spectacular, ABSOLUTE BEST strip of the playing field to make the shot. You can find our results above. This is how we did it:

| We started with a few basic assumptions; | |
|---|---|
| The radius of the ball "r"= 12 inches | Given. |
| The top of the goal "$G_{max}$"= 119.75 inches | Given. |
| The bottom of the goal "$G_s$"= 82.75 inches | Given. |
| Our shooting angle "α"=> (30 ≤ α ≥ 45) degrees | Common sense. |
| Our maximum range of distances "$R_s$" ≤ 162 inches | We don't want to waste recourses getting our robot to work harder than it needs to, and at 162 inches we have the entire last third of the field including the width of our robot to shoot from, which is the space where the most points will be made considering that it is the place where the final of three passes will be made. |
| Our initial distance from the ground "Yi" = 43.325 inches | Given. Give or take a negligible 1.537 inches depending on what angle we shoot from. |
| velocity "V"= d/t | Given. By definition. |


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| acceleration due to gravity "a"=V/t=386.088in/sec$^2$ | Given. Physics. |
| -Initial velocity: $V = \sqrt{V_x^2 + V_y^2}$ | Given. Where Vx= initial "x" component, and Vy= initial "y" component. |
| $V_{avg} = \Delta V/2$ | Given. Average velocity. |

| | |
|---|---|
| Then extrapolated some more to make things easier on ourselves; | |
| Finding the Lowest point in trajectory to make the goal, "$Y_s$"; | |
| $Y_s = G_s + r - Y_i$ | Given. |
| $Y_s = 51.425\ inches$ | Substituted and solved. |
| Finding the apex of our trajectory, (To barely scrape the top of the goal.) "$Y_m$"; | |
| $Y_{max} = G_{max} - r - Y_i$ | Given. |
| $Y_{max} = 64.425\ inches$ | Substituted and solved. |

| | |
|---|---|
| Found the time, "tm", taken to reach our apex; | |
| $Y_t = Vt + \frac{1}{2}at^2$ | Given. Definition of position with accelleration. |
| $Y_{max} = V_y t + \frac{1}{2}at^2$ | Substituted. |
| $Y_{max} = (0)t + \frac{1}{2}at^2$ | Assumed $V_y$=0 when $Y_{max}$ is reached. |
| $Y_{max} = \frac{1}{2}at^2$ | Simplified. |
| $t_{max} = \sqrt{2Y_{max}/a}$ | Isolated. |
| $t_{max} = 0.5777\ seconds$ | Substituted and solved. |

| | |
|---|---|
| Found the initial velocity in the "y" direction, "$V_y$", in order to reach our apex; | |
| $Y_t = Vt + \frac{1}{2}at^2$ | Given. |
| $V_t = V + at$ | Derived for velocity at time. |
| $V_{y_t} = V_y + at$ | Substituted. |
| $0 = V_y + at_{max}$ | $V_{yt}$=0 at $t_{max}$. |
| $V_y = -at_{max}$ | Isolated |
| $V_y = 223.041\ in/sec$ | Substituted and solved. |


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| Found our initial "x" velocity, "$V_x$", as determined by "$V_y$"; | |
|---|---|
| $\frac{\sin A}{a} = \frac{\sin B}{b}$ | Law of sines. |
| $\frac{\text{Sin}\,\alpha}{V_y} = \frac{\sin(90-\alpha)}{V_x}$ | Substituted and right triangle identity. |
| $\frac{\text{Sin}\,\alpha}{V_y} = \frac{\cos\alpha}{V_x}$ | Simplified by identity. |
| $V_x = {V_y \cos\alpha}/{\sin\alpha}$ | Isolated. |
| $V_x = 223.041 \cot\alpha$ | Simplified and substituted. |

| Found the time taken to reach "$Y_s$"; | |
|---|---|
| $Y_t = V_y t - \frac{1}{2}at^2$ | Given. Physics. |
| $0 = -\frac{1}{2}at^2 + V_y t - Y_t$ | Isolated. |
| $0 = -\frac{1}{2}a{t_s}^2 + V_y t_s - Y_s$ | Substituted. |
| $t_s = \frac{-V_y \pm \sqrt{{V_y}^2 - 4(-\frac{1}{2}a)(-Y_s)}}{-a}$ | Applied quadratic formula. |
| $t_s = 0.3183\ seconds\ and\ 0.8371\ seconds$ | Substituted and solved. |

| Found the time taken, "$t_f$", to return to its original "y" position, "$Y_i$", at "$X_f$" ; | |
|---|---|
| $t_f = 2t_{max}$ | Given. Parabolas are symmetrical. |
| $t_f = 1.1554\ seconds$ | Substituted and solved. |

| And finally found our best range of distances, "$R_s$", of angle, "α"; | |
|---|---|
| $\text{R}_\text{s} = X_f - 2X_s$ | Given. |
| $X_f = t_f V_x$ | Given. |
| $X_s = t_s V_x$ | Given. |
| 30 ≤ α ≥ 45 | Given. |
| $R_s$ ≤ 162 inches | Given. |
| $R_s = t_f V_x - 2t_s V_x$ | Substituted. |
| $R_s = (1.1554)(223.041)\cot\alpha - 2(0.3183)(223.041)\cot\alpha$ | Substituted. |
| $R_s = 115.7137 \cot\alpha$ | Simplified. |
| $R_s' = -115.7137 \csc\alpha \cot\alpha$ | Derived twice. |


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| | |
|---|---|
| $0 = -115.7137 \csc\alpha \cot\alpha$ | Set to zero. |
| $R_s = 162 inches\ when\ \alpha = \sim 39\ degrees$ (Rounded to nearest degree.) | Solved, constrained to "30 ≤ α ≥ 45" and "$R_s$ ≤ 162 inches", and chose the middle of 3 values to give a balanced minimum distance from the goalie zone! |

| Initial Velocity of Ball, "$V_i$", given optimum angle, "α"; | |
|---|---|
| $V_i = \sqrt{{V_x}^2 + {V_y}^2}$ | Given. Physics. |
| $V_i = \sqrt{(223.041 \cot 39)^2 + 223.041^2}$ | Substituted. |
| $V_i = \sim 354.42 \frac{in}{sec}\ or\ \sim 29.53 \frac{ft}{sec}$ | Solved. |

No doubt you will want someone who enjoys rigorous mathematics suiting these calculations to your own robot, but it's worth the work, as this not only draws a bold line between a **good** shooter and a bad shooter, but also marks the difference between a good shooter and a **perfect** shooter.

Graph of trajectory of ball at 39 degrees and 29.53 ft/sec:


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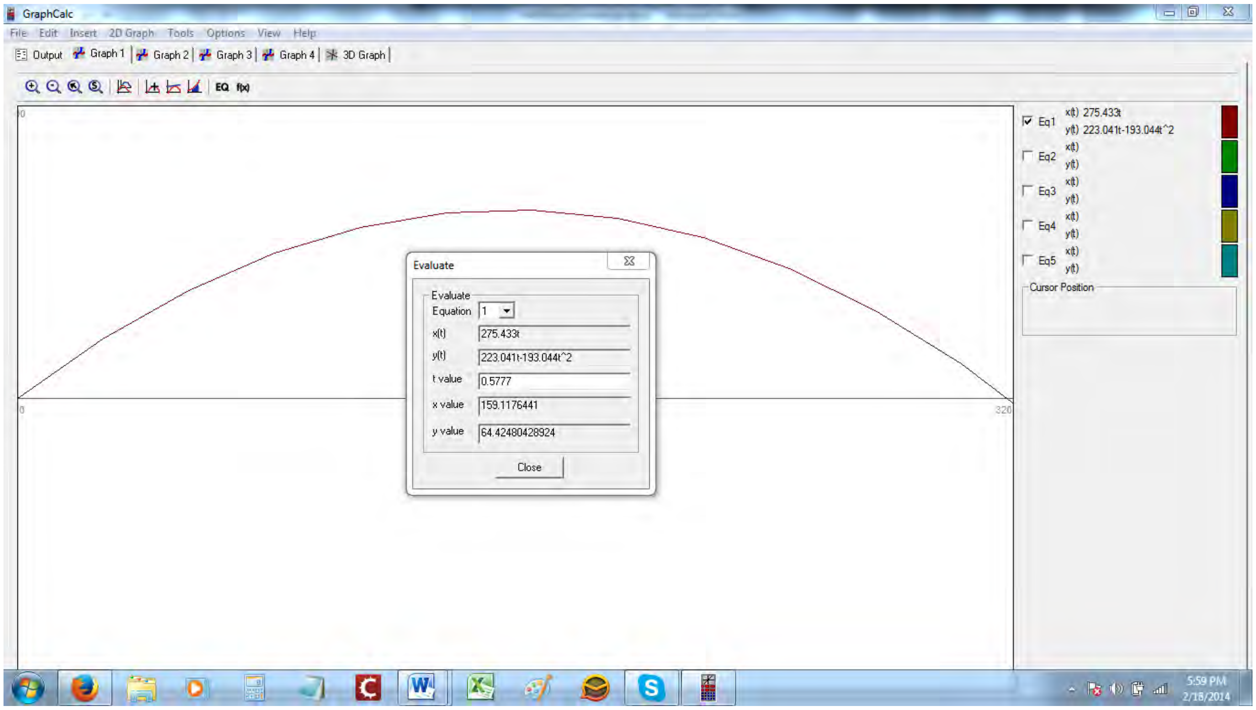
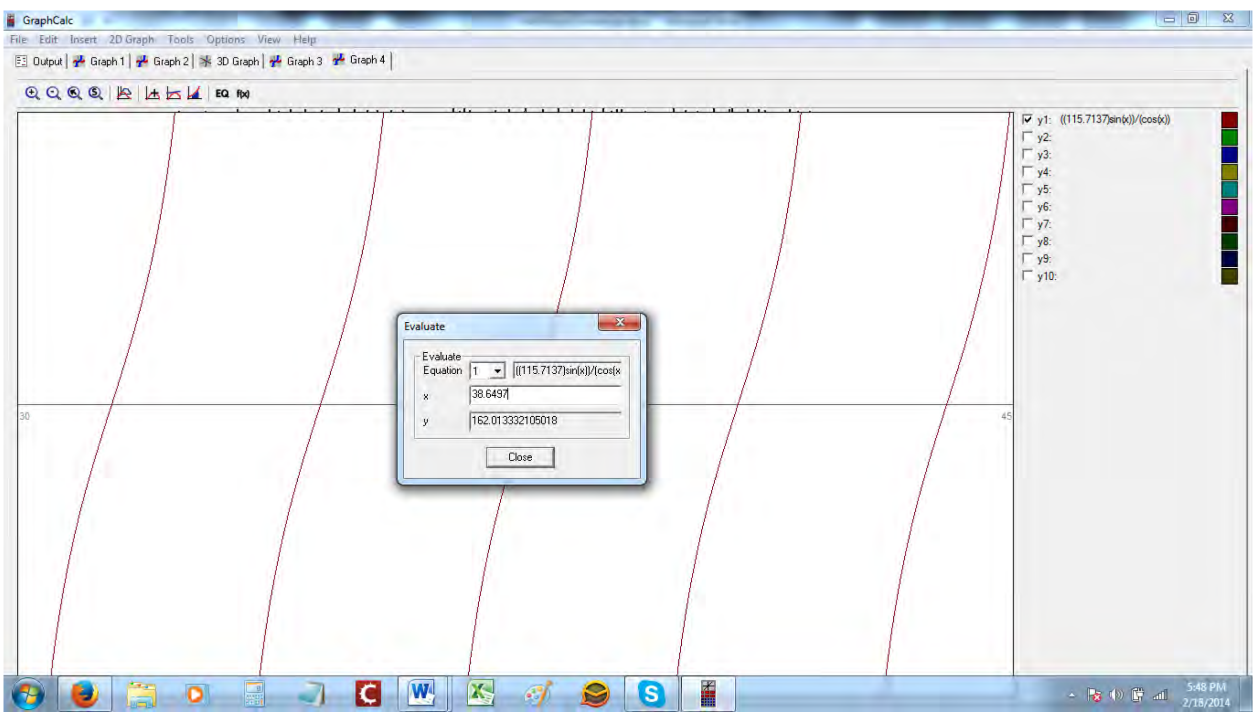



Graph of how our viable range of distances grows and shrinks as a function of our angle:





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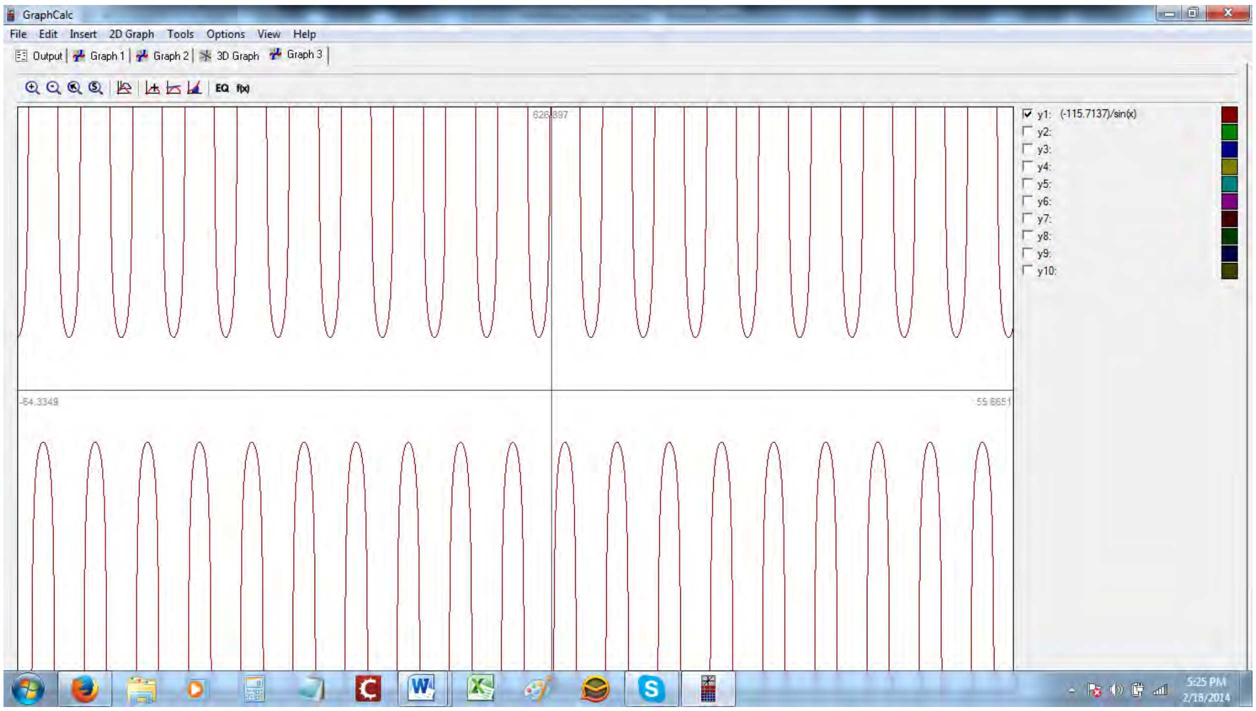
# STEMpunk's shooter, optimized with applied mathematics:

First derivative of above function:




Second derivative to find absolute maximum by finding a point at which the function equals zero: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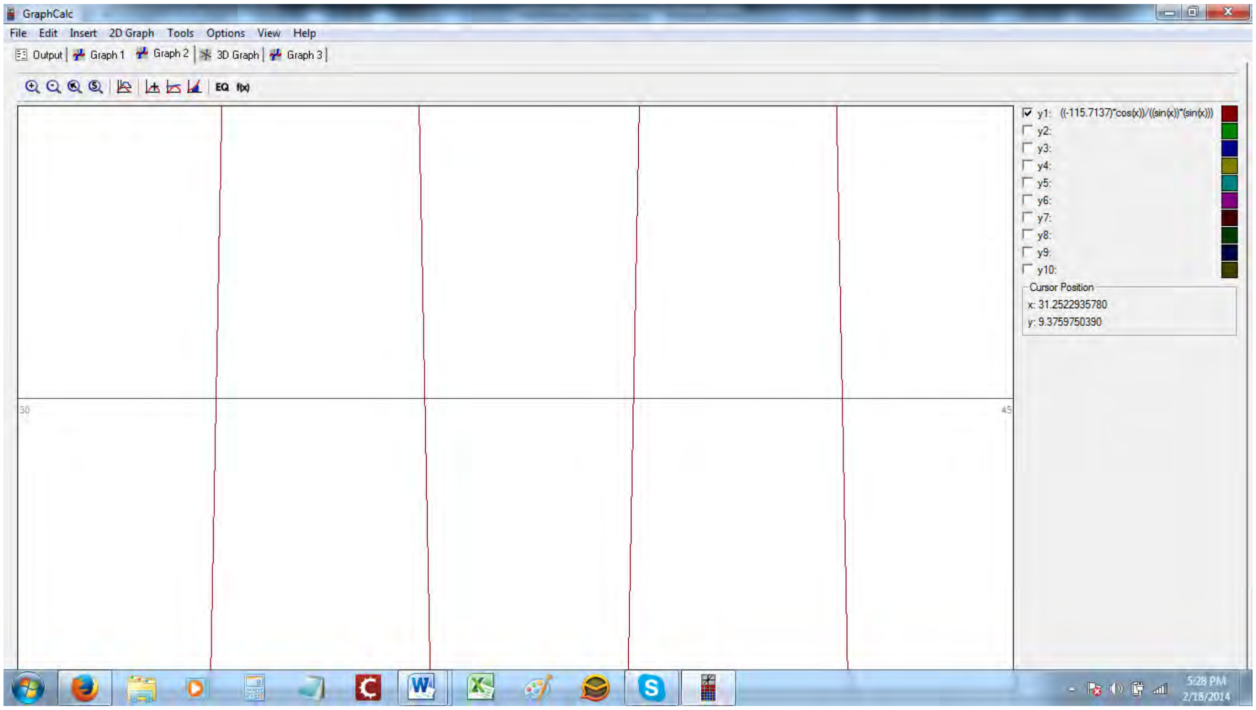



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